

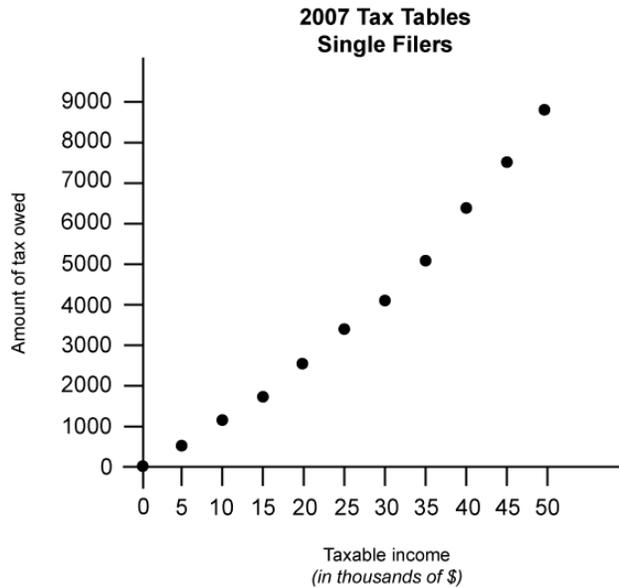
## Using Recursion in Models and Decision Making: Relationships in Data

### IV.A Student Activity Sheet 1: Using Scatterplots in Reports

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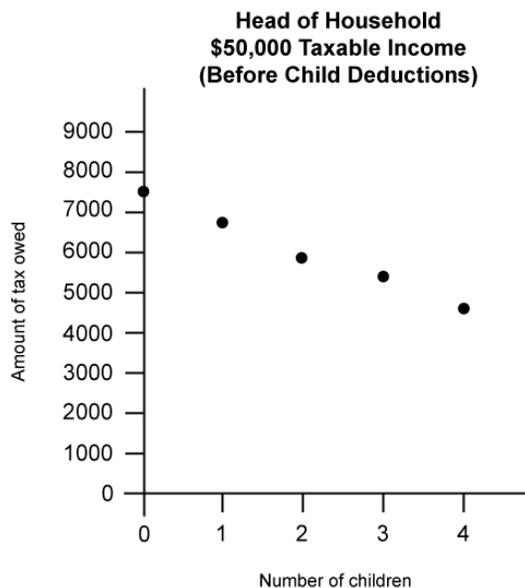
1. Consider the following graph. Who are the subjects in the study? What are the variables of interest?

Thoroughly describe the information illustrated by the graph, choosing at least two data points to help with your explanation.



(Data compiled from 2007 tax tables for Form 1040 on [www.irs.gov](http://www.irs.gov).)

2. Look at this new graph and discuss with your partner the information illustrated. Then compare and contrast this display with the graph in Question 1.



(Data compiled from 2007 tax tables for Form 1040 on [www.irs.gov](http://www.irs.gov).)

## Using Recursion in Models and Decision Making: Relationships in Data

### IV.A Student Activity Sheet 1: Using Scatterplots in Reports

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3. **REFLECTION:** Use the previous graphs to complete the following sentences.

A person with higher taxable income pays \_\_\_\_\_.

A person with lower taxable income pays \_\_\_\_\_.

This is an example of a \_\_\_\_\_ association.

A person with fewer children pays \_\_\_\_\_.

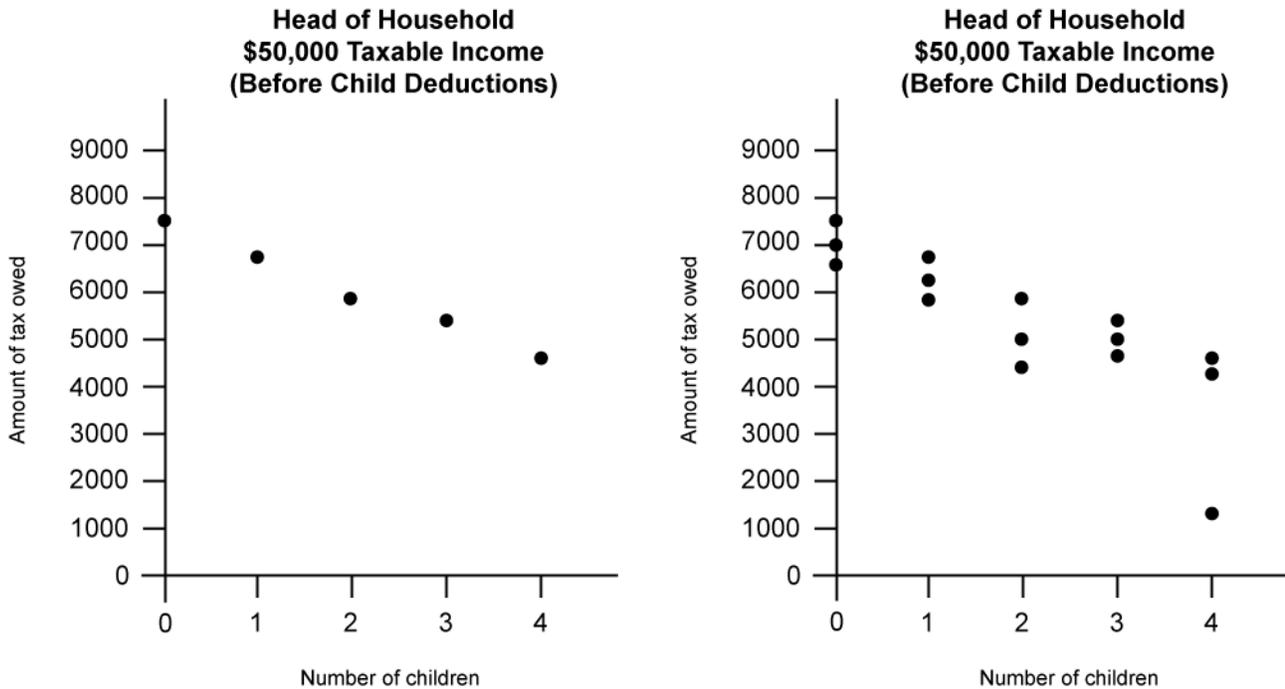
A person with more children pays \_\_\_\_\_.

This is an example of a \_\_\_\_\_ association.

## Using Recursion in Models and Decision Making: Relationships in Data

### IV.A Student Activity Sheet 1: Using Scatterplots in Reports

4. In actuality, head-of-household filers with \$50,000 in taxable income and the same number of children could pay different amounts of income tax, as shown by the graph on the right. These differences result from tax credits for expenses such as child care that can reduce the amount of tax owed. Compare and contrast this new graph with the original on the left.

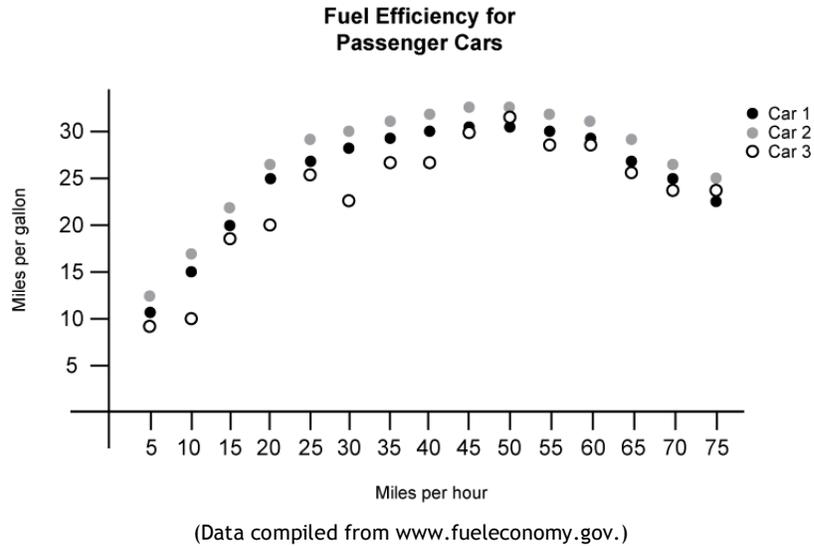


(Data compiled from 2007 tax tables for Form 1040 on [www.irs.gov](http://www.irs.gov).)

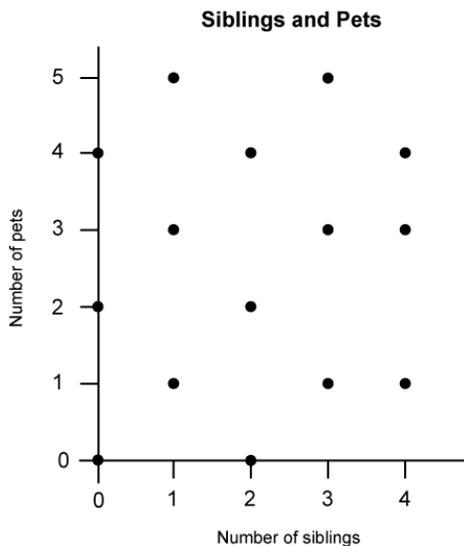
## Using Recursion in Models and Decision Making: Relationships in Data

### IV.A Student Activity Sheet 1: Using Scatterplots in Reports

5. Now consider the following graph. What information is displayed? Compare and contrast this graph with the others you have analyzed.



6. A survey of students asked, “How many siblings live in your house with you?” and “How many pets does your family have?” The results are displayed below. Comment on the graph, comparing and contrasting it with the previous graphs.



## Using Recursion in Models and Decision Making: Relationships in Data

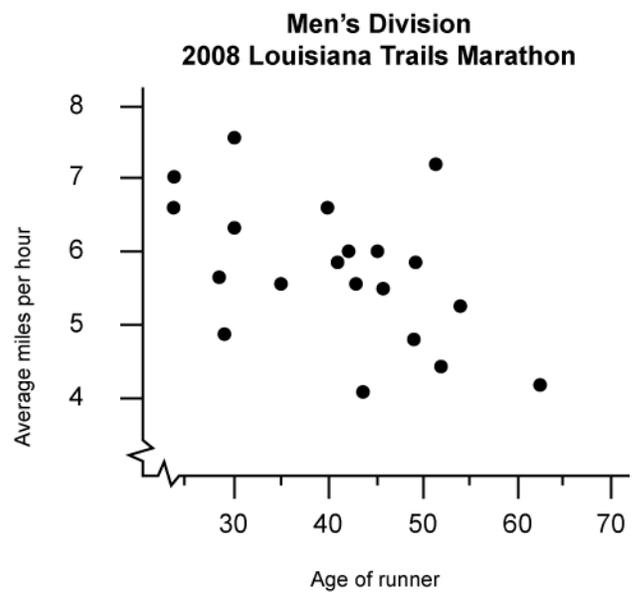
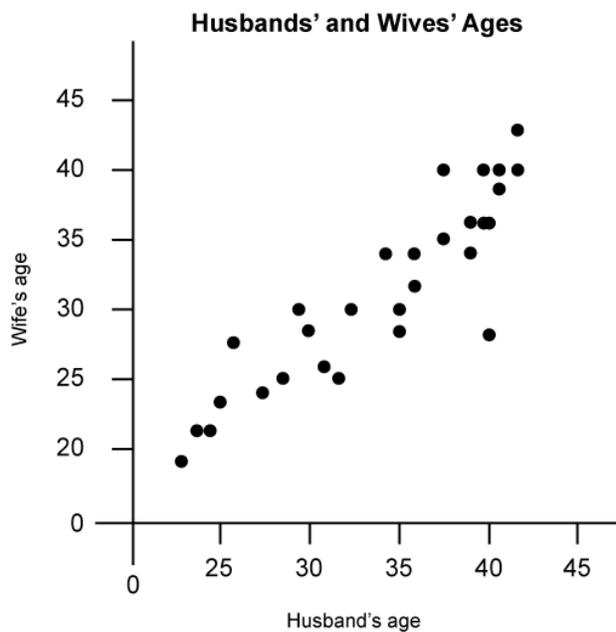
### IV.A Student Activity Sheet 1: Using Scatterplots in Reports

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7. When analyzing a display of bivariate statistics, you need to consider the following:

- Form—Does the graph exhibit a linear or nonlinear pattern?
- Direction—Does the graph exhibit a positive relationship, a negative relationship, or neither?
- Relative strength—Are the data points tightly clustered along the line or curve (strongly associated) or are they more scattered (weakly associated)?

Using these guidelines, analyze the following graphical displays. Conduct your analysis in the context of the situation.

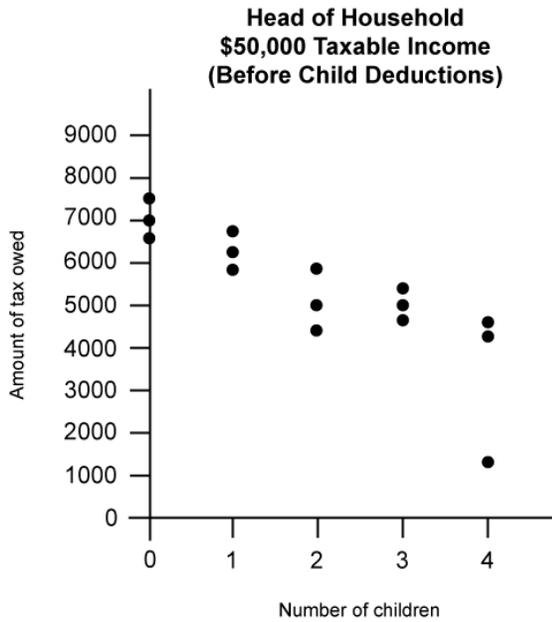


(Data adapted from marathonguide.com.)

## Using Recursion in Models and Decision Making: Relationships in Data

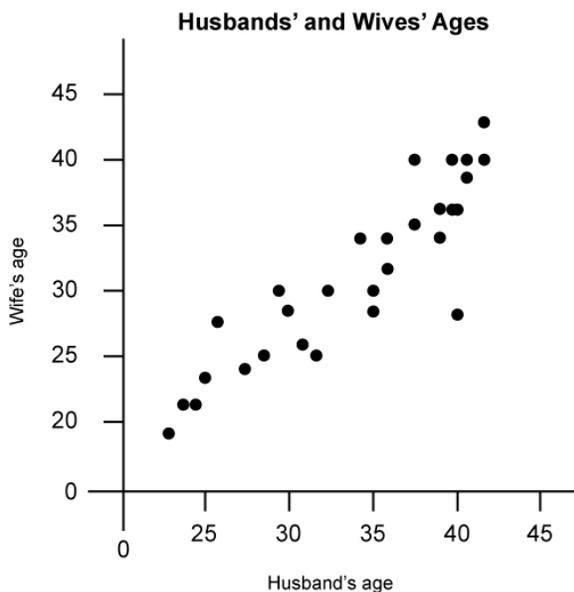
### IV.A Student Activity Sheet 1: Using Scatterplots in Reports

8. The following graph illustrates the fact that for a designated filing status and taxable income level, the amount of tax owed depends on the number of children. Does this sound like a cause-and-effect relationship or simply a matter of an association between the variables?



(Data compiled from 2007 tax tables for Form 1040 on [www.irs.gov](http://www.irs.gov).)

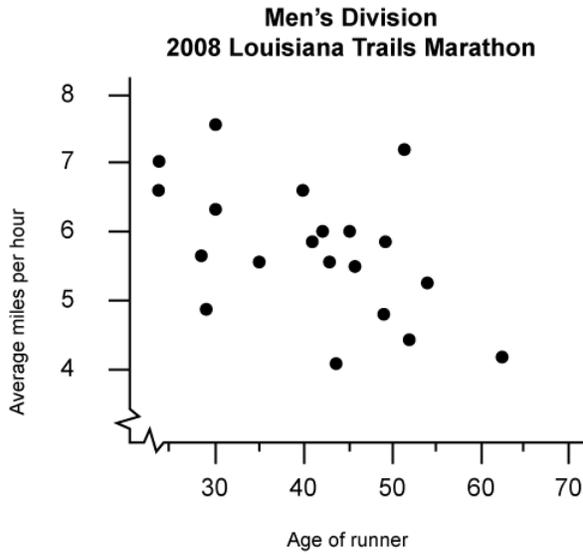
9. The graph shown below illustrates that in general older men have wives about their age and younger men have wives about their age. Does this sound like a cause-and-effect relationship or simply a matter of an association between the variables?



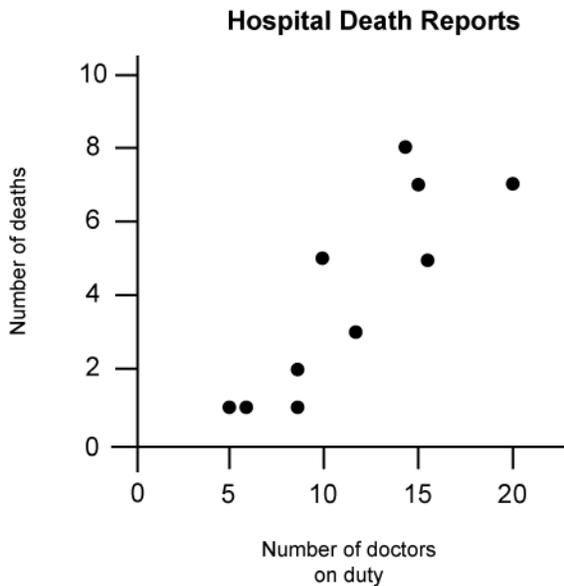
**Using Recursion in Models and Decision Making: Relationships in Data**  
 IV.A Student Activity Sheet 1: Using Scatterplots in Reports

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10. **EXTENSION:** A news report noted, “As men age, they begin to run slower.” Does this report imply cause and effect or association? What is your opinion of this implication?



11. **EXTENSION:** A special report on the evening news exposed a startling fact: When more doctors are on duty at a hospital, more deaths occur. Does this mean that doctors are killing patients? What are some other explanations?



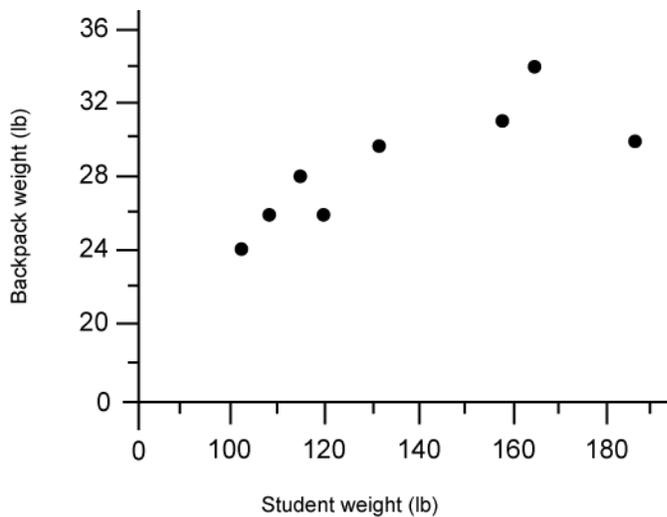
## Using Recursion in Models and Decision Making: Relationships in Data

### IV.A Student Activity Sheet 1: Using Scatterplots in Reports

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12. **EXTENSION:** In the last 40 years, spending on education has increased, while SAT scores have gone down. Sketch a scatterplot that represents these trends. Does increased spending cause a drop in SAT scores? Explain your reasoning.

13. **EXTENSION:** Doctors have become concerned about the effect of backpack weights on students' backs. Studies showed that, in general, students weighing more carry heavier backpacks. Write an analysis of the situation for the school newspaper. Clearly indicate in the analysis whether this is a situation of cause and effect or association.



## Using Recursion in Models and Decision Making: Relationships in Data

### IV.A Student Activity Sheet 1: Using Scatterplots in Reports

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14. **EXTENSION:** In the computer lab, conduct searches for examples of causation and correlation. If you find a misleading report, write a new report that clarifies the issue. If you cannot find a misleading report, describe how such a report might be written and how you would improve on it.

*Responses to this Extension will be unique to students' thinking and reasoning skills. Students may communicate their answers in a variety of ways.*

## Using Recursion in Models and Decision Making: Relationships in Data

### IV.A Student Activity Sheet 2: Recursion and Linear Functions

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1. Coen decides to take a job with a company that sells magazine subscriptions. He is paid \$20 to start selling and then earns \$1.50 for each subscription he sells. Fill in the following table, showing the amount of money ( $M$ ) Coen earns for selling  $n$  subscriptions. Use the process column to note what is happening in each line.

$n$	Process	$M_n$
0		$M_0 =$
		$M_1 =$
		$M_2 =$
		$M_3 =$
		$M_4 =$

2. Write a recursive rule for the amount of money Coen can earn selling magazine subscriptions.

## Using Recursion in Models and Decision Making: Relationships in Data

### IV.A Student Activity Sheet 2: Recursion and Linear Functions

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3. **REFLECTION:** The rule in Question 2 defines a term ( $M_{n+1}$ ) with respect to the term that precedes it ( $M_n$ ). Write a rule that defines a term ( $M_n$ ) with respect to the term that precedes it ( $M_{n-1}$ )? How is this rule similar to and different from the rule you wrote in Question 2?
4. Write an explicit function rule for the  $n$ th term in the sequence describing the amount of money Coen can earn. Describe any domain restrictions in your rule. How is this rule related to the rules you wrote in Question 2?

## Using Recursion in Models and Decision Making: Relationships in Data

### IV.A Student Activity Sheet 2: Recursion and Linear Functions

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5. Use sequence notation to enter the data from your table in Question 1 in a graphing calculator, if your calculator has this capability. Limit your lists to 50 entries each. How do you expect the scatterplot of your data to look? Justify your reasoning.
  
  
  
  
  
  
  
  
  
  
6. How much does Coen earn if he sells 100 magazine subscriptions? Which rule did you use to answer this question? Why did you choose that rule?
  
  
  
  
  
  
  
  
  
  
7. Coen is trying to earn enough money to buy a new MP3 player. He needs \$225 to cover the cost and tax on the MP3 player. How many magazine subscriptions does Coen need to sell to buy his new MP3 player? Justify your answer. Which rule did you use to answer this question? Why did you choose that rule?

## Using Recursion in Models and Decision Making: Relationships in Data

### IV.A Student Activity Sheet 2: Recursion and Linear Functions

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8. Your phone service allows you to add international long distance to your phone. The cost is a \$5 flat fee each month and 3¢ a minute for calls made. Write a recursive rule describing your monthly cost for international calls. Then write a function rule for the  $n$  minutes of calls made in a month.

### 9. REFLECTION

- How are recursive rules different from explicit function rules for modeling linear data?
- How are they the same?
- When are recursive rules more useful than function rules?
- When are function rules more useful?

## Using Recursion in Models and Decision Making: Relationships in Data

### IV.A Student Activity Sheet 2: Recursion and Linear Functions

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10. **EXTENSION:** Think of a situation that can be described by a linear function. Model the situation using a recursive rule and a function rule. Write a question that is better answered using the recursive rule and give the solution.

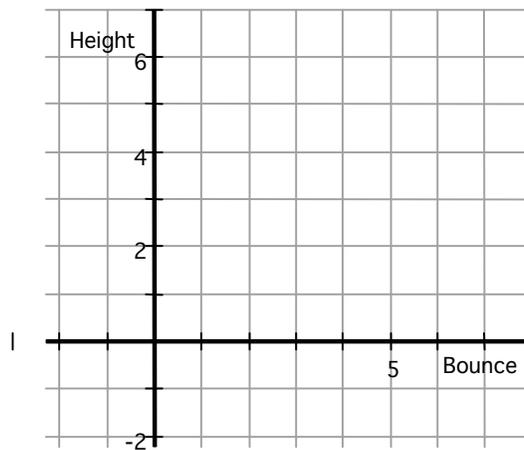
## Using Recursion in Models and Decision Making: Recursion in Exponential Growth and Decay

### IV.B Student Activity Sheet 3: Recursion and Exponential Functions

Different balls bounce at various heights depending on things like the type of ball, the pressure of air in the ball, and the surface on which it is bounced. The rebound percentage of a ball is found by determining the quotient of the rebound height (that is the height of each bounce) to the height of the ball before that bounce, converted to a percentage.

1. Collect data on a bouncing ball that show the maximum height of at least five bounces of the ball. Then make a scatterplot of the maximum height as a function of the bounce number. (Let Bounce 0 be the initial drop height of the ball.)

Bounce No.	Height
0	
1	
2	
3	
4	
5	



2. Find the average rebound percentage for your ball. Show your work.

Bounce No.	Height	Process	Rebound Percentage
0			
1			
2			
3			
4			
5			

## Using Recursion in Models and Decision Making: Recursion in Exponential Growth and Decay

### IV.B Student Activity Sheet 3: Recursion and Exponential Functions

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3. Tennis balls are sealed in a pressurized container to maintain the rebound percentage of the balls. A tennis ball has a rebound percentage of 55% when it is taken out of the pressurized can. Suppose a tennis ball is dropped from a height of 2 meters onto a tennis court. Use the rebound rate given to predict the height of the ball's first seven bounces.

Bounce No.	Process	Height (m)
0	(initial drop height given)	2
1		
2		
3		
4		
5		
6		
7		

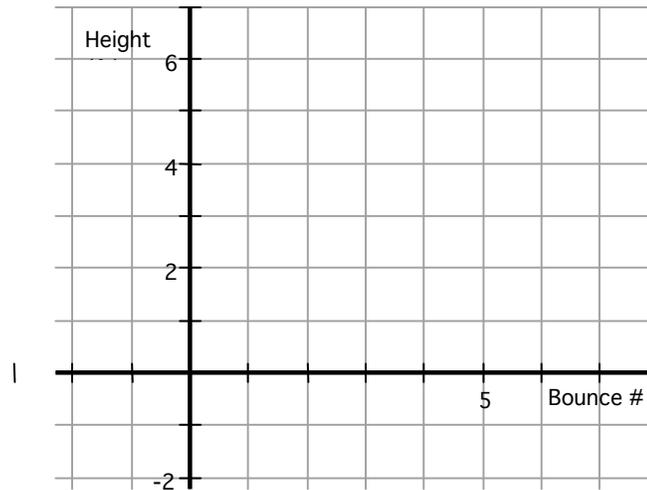
4. Write a recursive rule for the height of the ball for each successive bounce.
5. Describe, in words, how the height of each bounce is calculated from the height of the previous bounce.

## Using Recursion in Models and Decision Making: Recursion in Exponential Growth and Decay

### IV.B Student Activity Sheet 3: Recursion and Exponential Functions

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6. Enter the bounce height data into a graphing calculator. Make a scatterplot and then sketch the graph below.



7. What kind of function might model the tennis ball bounce situation? Explain your reasoning with a table of values or other representation.

## Using Recursion in Models and Decision Making: Recursion in Exponential Growth and Decay

### IV.B Student Activity Sheet 3: Recursion and Exponential Functions

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8. Look back at the table you generated in Question 3. Write a function rule for bounce height in terms of bounce number. Graph the function rule with the scatterplot on your graphing calculator to see if the function rule models the data.
9. What is the height of the fifth bounce of a new tennis ball if the initial drop height is 10 meters above the ground? Use a function rule to find your answer.

## Using Recursion in Models and Decision Making: Recursion in Exponential Growth and Decay

### IV.B Student Activity Sheet 3: Recursion and Exponential Functions

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10. Suppose a new tennis ball is dropped from a height of 20 feet. How many times does it bounce before it has a bounce height of less than 4 inches (the diameter of the ball)? Explain your solution.
11. What is the total vertical distance that the ball from Question 10 has traveled after six bounces? Explain your answer.
12. **REFLECTION:** How can you decide if a data set can be modeled by an exponential function? How are recursive rules different from function rules for modeling exponential data? How are they the same?

## Using Recursion in Models and Decision Making: Recursion in Exponential Growth and Decay

### IV.B Student Activity Sheet 3: Recursion and Exponential Functions

**13. EXTENSION:** As our population uses more antibiotics for minor infections, bacteria adapt and become resistant to the medications that are available. Methicillin-resistant *Staphylococcus aureus* (MRSA) is strain of *Staphylococcus* bacteria that is resistant to antibiotics. MRSA causes skin and respiratory infections and can be fatal. The Center for Disease Control (CDC) reported that in the United States 94,360 MRSA infections occurred in 2005, with 18,650 of these cases resulting in deaths. To put this number in perspective, CDC reported 16,316 deaths in 2005 related to AIDS.

A research lab is observing the growth of a new strain of MRSA in an agar dish. The initial area occupied by the bacteria is 2 square millimeters. In previous experiments with MRSA, scientists observed the bacteria to increase by 20% each week.

- Fill in the table, showing the increase in area of the bacteria over eight weeks.
- Write a recursive rule for the area ( $a$ ) of the bacteria after  $n$  weeks.
- Then make a scatterplot of the data.

Week	Process	Area (mm <sup>2</sup> )
0		2
1		
2		
3		
4		
5		
6		
7		
8		
$n$		



## Using Recursion in Models and Decision Making: Recursion in Exponential Growth and Decay

### IV.B Student Activity Sheet 3: Recursion and Exponential Functions

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#### 14. EXTENSION

- Research a set of population data. Use data from a country's census report or other state/national population data or animal population data.
- Cite the source of the data.
- Decide if the data, or a portion of the data, can be modeled by an exponential function.
- Support your decision with a mathematical argument.
- Find an appropriate model for the data.
- Based on your model, make a prediction about the population.



## Using Recursion in Models and Decision Making: Recursion in Exponential Growth and Decay

### IV.B Student Activity Sheet 4: Comparing Models

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3. In an effort to speed up the time needed to save \$2,000, Derrick decides to take on some jobs in his community. Suppose he commits to adding \$50 per month to his savings, starting with the initial deposit from his parents. Fill in the table, showing the amount of money Derrick will have over several months.

Months	Process	Dollars
0		200
1		
2		
3		
4		
5		
6		
7		
8		

4. Make a scatterplot of the data you generated in the table and compare the scatterplot to the function rule you found for Question 1. How does adding \$50 per month to Derrick's savings change the way in which his money grows?









## Using Recursion in Models and Decision Making: Recursion Using Rate of Change

### IV.C Student Activity Sheet 5: Newton's Law of Cooling

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9. How does your scatterplot compare to the graph you sketched at the beginning? Explain any differences.
10. How does your scatterplot support the type of relationship you chose in Question 5?
11. The general form for an exponential function is  $y = a(b)^x$ , where  $a$  represents the initial condition and  $b$  represents the successive ratio, or base of the exponential function. Using data from your table, write a function rule to describe the temperature of the coffee ( $y$ ) as a function of time ( $x$ ).
12. What do the constants in the function rule represent?

## Using Recursion in Models and Decision Making: Recursion Using Rate of Change

### IV.C Student Activity Sheet 5: Newton's Law of Cooling

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13. Graph the function rule over the data in your scatterplot. Sketch your results.

14. **REFLECTION:** Compare the recursive rule and explicit function rule that you wrote in the previous questions. What do you notice?

If you repeated this experiment in a room that was much cooler, what changes in your data would you expect? Why do you think so?

15. What would a scatterplot of the change in temperature ( $\Delta T$ ) versus the difference between the liquid's temperature and the room temperature ( $T$ ) look like? Sketch your prediction, if needed.

## Using Recursion in Models and Decision Making: Recursion Using Rate of Change

### IV.C Student Activity Sheet 5: Newton's Law of Cooling

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16. Use your graphing calculator to make a scatterplot of the change in temperature ( $\Delta T$ ) versus the difference between the liquid's temperature and the room temperature ( $T$ ). Sketch your graph.

How does your graph compare to your prediction?

What kind of function appears to model your graph?

## Using Recursion in Models and Decision Making: Recursion Using Rate of Change

### IV.C Student Activity Sheet 5: Newton's Law of Cooling

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17. **EXTENSION:** Use an appropriate regression routine to find a function rule to model your scatterplot of  $\Delta T$  versus  $T$ . Record your function rule, rounding to an appropriate number of places.

Graph the function rule over your scatterplot. How well does it fit the data?

## Using Recursion in Models and Decision Making: Recursion Using Rate of Change

### IV.C Student Activity Sheet 5: Newton's Law of Cooling

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Recall that a proportional relationship between the independent and dependent variables satisfies three criteria:

- The graph is linear and passes through the origin.
  - The function rule is of the form  $y = kx$ .
  - The ratio  $\frac{y}{x}$  is constant for all corresponding values of  $x$  and  $y$ .
18. **EXTENSION:** Is your function rule from Question 17 a proportional relationship? Defend your answer using all three criteria.

**Graph:**

**Function Rule:**

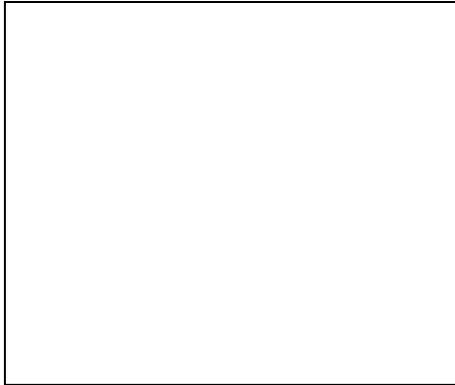
**Table:**

## Using Recursion in Models and Decision Making: Recursion Using Rate of Change

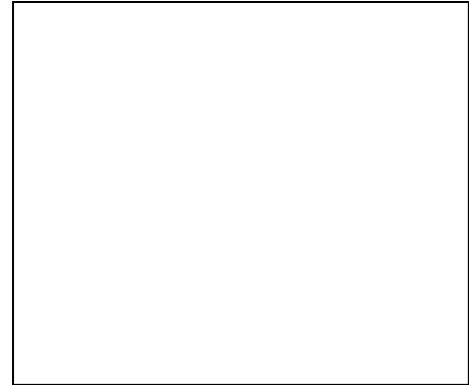
### IV.C Student Activity Sheet 5: Newton's Law of Cooling

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19. **REFLECTION:** Suppose the relationship between the change in temperature and the difference between the liquid's temperature and the ambient temperature is proportional. Write a proportionality statement to show the relationship between the two variables.



is proportional to



20. **EXTENSION:** Choose a simple exponential function to validate the proportionality according to all criteria.

## Using Recursion in Models and Decision Making: Recursion Using Rate of Change

### IV.C Student Activity Sheet 5: Newton's Law of Cooling

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21. **EXTENSION:** Use the Internet to research other situations that could be modeled using a function from the same family as the one for Newton's Law of Cooling. Obtain a data set and generate a function model. Cite your sources. Support your choice of function model with mathematical reasoning. Make a prediction from your data set using your model. How reasonable is your prediction?

## Using Recursion in Models and Decision Making: Recursion Using Rate of Change

### IV.C Student Activity Sheet 6: Rates of Change in Exponential Models

1. Consider the exponential function  $y = 2^x$ . Fill in the table of values for the function, and find the rate of change between consecutive values in the function ( $\Delta y$ ). What pattern do you see for  $\Delta y$  change?

$x$	$y$	$\Delta y$
0		
1		
2		
3		
4		
5		

2. In Student Activity Sheet 3, you learned about the sometimes fatal antibiotic-resistant staph bacteria methicillin-resistant *Staphylococcus aureus* (MRSA) growing in an agar dish. The initial area occupied by the bacteria in the agar dish is 2 square millimeters, and they increase in area by 20% each week. The table below gives the area of the bacteria over several weeks. Use the table to describe the rate of growth of the area of the bacteria ( $\Delta a$ ).

$x$ (no. of weeks)	$a$ (area in $\text{mm}^2$ )	$\Delta a$
0	2	
1	2.4	
2	2.88	
3	3.456	
4	4.147	
5	4.977	

## Using Recursion in Models and Decision Making: Recursion Using Rate of Change

### IV.C Student Activity Sheet 6: Rates of Change in Exponential Models

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3. Suppose a quantity increases at a rate proportional to the quantity, and the constant of proportionality is 0.2. The initial quantity is 2. Write a difference equation that describes the statement above, and find several values of this quantity.

$x$	$\Delta y$	$y$
0		2
1		
2		
3		
4		

4. Use spreadsheet software to generate about 75 values of the table you started in Question 3. The spreadsheet allows you to use a recursive rule to generate the data.

## Using Recursion in Models and Decision Making: Recursion Using Rate of Change

### IV.C Student Activity Sheet 6: Rates of Change in Exponential Models

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5. The agar dish that the MRSA bacteria are growing in has an area of 1,000 square millimeters. The growth of the bacteria is limited in the lab by the size of the agar dish. The growth of the bacteria can still be modeled by a proportional difference equation, but now the rate of increase of the bacteria's area is directly proportional to the bacteria's area and the difference between the agar dish's area and the fungus's area. This constant of proportionality is the ratio of the original constant of proportionality (0.2) and the maximum area the bacteria can reach (1,000 square millimeters). The difference equation can be written as follows:

$$\frac{\Delta A}{\Delta t} = \frac{0.2}{1,000} A(1,000 - A)$$

Let  $\Delta t = 1$  to simplify your work, and then use the difference equation to find the new values of  $A$ . Use a spreadsheet to calculate about 75 values.

6. Compare the data generated in the unrestricted and restricted growth models. Record your observations.

## Using Recursion in Models and Decision Making: Recursion Using Rate of Change

### IV.C Student Activity Sheet 6: Rates of Change in Exponential Models

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7. Use spreadsheet software or your graphing calculator to make a graph of the restricted growth model. Sketch the graph below. What observations can you make about the graph?

8. **REFLECTION:** The unrestricted bacteria growth models exponential growth and has a common ratio of 1.2. Use the spreadsheet to find the ratio between successive values in the restricted growth model. What do you notice? How does this support the graph in Question 7?

## Using Recursion in Models and Decision Making: Recursion Using Rate of Change

### IV.C Student Activity Sheet 6: Rates of Change in Exponential Models

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9. A rancher has decided to dedicate a 400-square-mile portion of his ranch as a black bear habitat. Working with his state, he plans to bring 10 young black bears to the habitat in an effort to grow the population. His research shows that the annual growth rate of black bears is about 0.8. Black bears thrive when the population density is no more than about 1.5 black bears per square mile.
- What is the maximum sustainable number of black bears for the habitat?
  - Write a recursive rule showing the restricted growth in population for the black bears. (*Hint:* The constant of proportionality is the ratio of the unrestricted growth rate and the maximum sustainable population.)
  - Make a table and graph showing the yearly population of the black bears in the habitat. (Include enough years to show the population reaching the maximum sustainable population.)

## Using Recursion in Models and Decision Making: Recursion Using Rate of Change

### IV.C Student Activity Sheet 6: Rates of Change in Exponential Models

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- d. When will the population of bears in the habitat reach 500?
- e. The rancher wants to repopulate the state with black bears. The rancher's original plan was to release the bears from his ranch when the population reaches 500. Do you think this is a good decision based on the growth rate within the habitat over time? If you agree with the rancher, support the decision with your data and graph. If you disagree, propose a different target population value to the rancher; again, support your proposal with the data and graph.

## Using Recursion in Models and Decision Making: Recursion Using Rate of Change

### IV.C Student Activity Sheet 6: Rates of Change in Exponential Models

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10. **EXTENSION:** Research population data, either of humans in various parts of the world or animal species. You need to find data over a significant time, not just a few years. Cite your source. Make a scatterplot of the data. Do the data show exponential growth or do they show signs that the population's growth is slowing? What limitations does the population you are analyzing have? Could you predict a maximum population? Support your prediction.

## Using Recursion in Models and Decision Making: Recursion in Cyclical Models

### IV.D Student Activity Sheet 7: Modeling the Singapore Flyer

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On February 11, 2008, Singapore opened a new observation wheel called the Singapore Flyer. At the time of its opening, this giant Ferris wheel was the tallest in the world. The Singapore Flyer consists of an observation wheel with a diameter of 150 meters atop a boarding terminal, giving the structure an overall height of 165 meters. Twenty-eight air-conditioned capsules rotate on the outside of the wheel to provide unobstructed views of the city. The wheel rotates at a constant rate of 26 centimeters per second. This is slow enough that the wheel does not need to stop for loading and unloading unless there are special passenger needs.



1. Using graph paper, draw an accurate diagram of the wheel showing the dimensions given above. Use a compass or other tool to accurately draw the circle.

## Using Recursion in Models and Decision Making: Recursion in Cyclical Models

### IV.D Student Activity Sheet 7: Modeling the Singapore Flyer

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2. On the next page, fill in the table showing the height of a single capsule changing as it rotates counterclockwise from the boarding terminal around the wheel. To do this, calculate the circumference of the wheel.
- How many minutes does it take a capsule to make one complete revolution around the wheel? (Round to the nearest minute.) Explain your process.
  - Before completing the table, explain how the angle values provided in the table are correct.
  - The first inscribed angle that models the situation after one minute is shown. Use additional diagrams and inscribed right triangles to determine more values of the total height as a given capsule continues to rotate through one complete revolution. Use trigonometry to calculate the corresponding values of  $a$  and complete the table, finding the height of the capsule at the various intervals of time.

**Using Recursion in Models and Decision Making: Recursion in Cyclical Models**  
 IV.D Student Activity Sheet 7: Modeling the Singapore Flyer

Time (min)	Angle of Rotation from Boarding Station	Vertical Leg of Right Triangle ( $a$ )	Process (If you decide to change your process, explain your decision.)	Total Height of Capsule (m)
1	$12^\circ$			
30	$360^\circ$			

3. Create a graph showing the height changing as a given capsule rotates through one complete revolution of the wheel. Show at least 10 well-spaced data points on your graph.



## Using Recursion in Models and Decision Making: Recursion in Cyclical Models

### IV.D Student Activity Sheet 7: Modeling the Singapore Flyer

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#### 7. EXTENSION

- Research another gigantic Ferris wheel. Find the dimensions of the wheel and how long it takes for the wheel to complete one revolution. Using that information, make a rough sketch of the height of a capsule over time.

OR

- What other situations can you think of that repeat themselves and could possibly be modeled with a periodic function? (Some examples include volume of air in lungs over time during rest, a pendulum swinging, the bounce of a *spring*, sea levels affected by tides, and sound waves.)